ISSN: 2639-0426

madridge PUBLISHERS
Interconnecting Scientific World

International Journal of Physics: Study and Research

Research Article

Open Access

Modified Dirac Equation Solutions and the Collapse of White Dwarfs and Neutron Stars

Thomas B. Fowler

George Mason University, USA

Article Info

***Corresponding author:**
Thomas B. Fowler
Adjunct Professor
Department of Electrical and Computer Engineering
George Mason University
Fairfax, VA
USA
E-mail: tbfowler@caa.columbia.edu

Received: March 11, 2024
Accepted: April 05, 2024
Published: April 12, 2024

Citation: Fowler TB. Modified Dirac Equation Solutions and the Collapse of White Dwarfs and Neutron Stars. *Int J Phys Stud Res*. 2024; 6(1): 112-118.
doi: 10.18689/ijpsr-1000119

Published by Madridge Publishers

Abstract

White Dwarf stars and Neutron stars both undergo catastrophic collapse under extreme conditions. The empirical fact to be explained is the sudden collapse of these stars when a certain combination of conditions involving pressure, spacetime curvature, and possibly temperature occurs. The account usually given is based on loss of degeneracy pressure, but does not explain the mechanism. In this paper, the proposed explanation of the fact is that under these conditions, fermions (electrons) transform into bosons, thereby ceasing to be governed by the Pauli Exclusion Principle, which accounts for degeneracy pressure. With no degeneracy pressure, atoms collapse and the White Dwarf then collapses into a Neutron star. A similar collapse can occur in the case of Neutron stars, when neutrons transform into bosons. The implication is that the electrons now form a new state of matter, one that is not normally seen and cannot be created in earthbound laboratories. This research combines well-known results from Quantum Field Theory and General Relativity, proposing a new, slightly modified version of the usual solutions to the Dirac equations to show how electrons or neutrons can transform from fermions to bosons under extreme conditions, to explain the degeneracy pressure failure. It is then used to explain the origin of the extreme magnetic fields of magnetars, and to give a possible identification of a superpartner for electrons.

Keywords: Dirac equation, Black Hole, White Dwarf, boson, fermion, Neutron Star, Supersymmetry.

1. Introduction

The collapse of White Dwarf stars to Neutron stars, and the further collapse of Neutron stars to Black Holes, poses several interesting and important questions about the behavior of matter under extreme conditions. A White Dwarf reaches a terminal size wherein it is prevented from collapsing by electron degeneracy pressure arising from the Pauli Exclusion Principle (Lequeux, 2013). This principle acts by preventing fermions from occupying arbitrary energy levels in dense configurations, such as solid state matter. But the principle only applies to fermions, not bosons. The defining characteristic of fermions is non-integral spin: for electrons, it is ½. Likewise, the defining characteristic of bosons is integral spin; for photons, it is 1. Can we understand transition of White Dwarfs sustained by electron degeneracy to neutron stars as a transition from fermions to bosons? This suggests that if we can modify the Dirac equation solutions so that under normal circumstances they are essentially unchanged, but under extreme circumstances the particles represented change from fermions to bosons, we can explain the collapse of White Dwarfs and Neutron stars. This requires the spin of the particles to become the same and to become an integer value.

2. Modification of Dirac Equation Solutions

The Dirac Equation takes the formshown in equation (1) (Schwichtenberg, 2020; Klauber, 2013):

$$i\partial_\mu \gamma^\mu \Psi - m\Psi = 0 \tag{1}$$

The base Dirac equation solutions for particles are given by equations (2) and (3):

$$\left|\Psi^{(1)}\right\rangle = \sqrt{\frac{E+m}{2m}} \begin{bmatrix} 1 \\ 0 \\ \frac{p^3}{E+m} \\ \frac{p^1+ip^2}{E+m} \end{bmatrix} e^{-ipx} \quad \left|\Psi^{(2)}\right\rangle = \sqrt{\frac{E+m}{2m}} \begin{bmatrix} 0 \\ 1 \\ \frac{p^1-ip^2}{E+m} \\ \frac{-p^3}{E+m} \end{bmatrix} e^{-ipx} \tag{2}$$

Spin up Spin down

And for the corresponding anti-particles:

$$\left|\Psi^{(3)}\right\rangle = \sqrt{\frac{E+m}{2m}} \begin{bmatrix} \frac{p^3}{E+m} \\ \frac{p^1+ip^2}{E+m} \\ 1 \\ 0 \end{bmatrix} e^{-ipx} \quad \left|\Psi^{(4)}\right\rangle = \sqrt{\frac{E+m}{2m}} \begin{bmatrix} \frac{p^1-ip^2}{E+m} \\ \frac{-p^3}{E+m} \\ 0 \\ 1 \end{bmatrix} e^{-ipx} \tag{3}$$

Spin up Spin down

The object of the solution modifications is to determine a new version of these solutions that would give boson behavior to fermions under extreme spacetime curvature, i.e., account for electron and neutron behavior in stars. We do this by making an appropriate adjustment to each solution term such that under normal conditions (little spacetime curvature) there is no measurable or observable change from current solutions. But under extreme spacetime curvature, there is a significant nearly step function type of change. Of course, the modified solutions must continue to satisfy the original Dirac equation. The new solutions, where *k* = some function associated with spacetime curvature, for spin up particles is given by equation (4):

$$\left|\Psi^{(1)}\right\rangle^* = \sqrt{\frac{E+m}{2m}} \begin{bmatrix} 1-0.5/e^{1/k} \\ 0.5/e^{1/k} \\ 0.5\frac{p_1-ip_2-p_3}{e^{1/k}(E+m)} + \frac{p_3}{E+m} \\ \frac{p_1+ip_2}{E+m} - 0.5\frac{p_1+ip_2+p_3}{e^{1/k}(E+m)} \end{bmatrix} e^{-i(Et-px)} \tag{4}$$

And corresponding equation for spin down given by equation (5):

$$\left|\psi^{(2)}\right\rangle^* = \sqrt{\frac{E+m}{2m}} \begin{bmatrix} 0.5/e^{1/k} \\ 1-0.5/e^{1/k} \\ \frac{p_1-ip_2}{E+m} - 0.5\frac{p_1-ip_2-p_3}{e^{1/k}(E+m)} \\ -\frac{p_3}{E+m} + 0.5\frac{p_1+ip_2+p_3}{e^{1/k}(E+m)} \end{bmatrix} e^{-i(Et-px)} \tag{5}$$

For $k \to 0$, this reduces to the original Dirac solutions, as expected. For $k \to \infty$ both of these solutions reduce to equation (6):

$$\lim_{k\to\infty}\left|\psi^{(1)}\right\rangle = \lim_{k\to\infty}\left|\psi^{(2)}\right\rangle = \frac{1}{2}\sqrt{\frac{E+m}{2m}} \begin{bmatrix} 1 \\ 1 \\ \frac{p_1-ip_2+p_3}{(E+m)} \\ \frac{p_1+ip_2-p_3}{(E+m)} \end{bmatrix} e^{-i(Et-px)} \tag{6}$$

In this case spin up and spin down solutions become the same. The only way that this can happen is if spin is 0. Integer spins are associated with bosons, which show that you can get boson behavior from particles described by the Dirac equations without modifying them. We will denote the boson form of electrons as e_b^-. For anti-particles (positrons in the case of electrons), the new solutions take the form of equations (7) and (8):

$$\left|\Psi^{(3)}\right\rangle = \sqrt{\frac{E+m}{2m}} \begin{bmatrix} 0.5\frac{p_1-ip_2-p_3}{e^{1/k}(E+m)} + \frac{p_3}{E+m} \\ \frac{p_1+ip_2}{E+m} - 0.5\frac{p_1+ip_2+p_3}{e^{1/k}(E+m)} \\ 1-0.5/e^{1/k} \\ 0.5/e^{1/k} \end{bmatrix} e^{-i(Et-px)} \tag{7}$$

$$\left|\Psi^{(4)}\right\rangle = \sqrt{\frac{E+m}{2m}} \begin{bmatrix} \frac{p_1-ip_2}{E+m} - 0.5\frac{p_1-ip_2-p_3}{e^{1/k}(E+m)} \\ 0.5\frac{p_1+ip_2+p_3}{e^{1/k}(E+m)} - \frac{p_3}{E+m} \\ 0.5/e^{1/k} \\ 1-0.5/e^{1/k} \end{bmatrix} e^{-i(Et-px)} \tag{8}$$

It is easily shown that these reduce to the original anti-particle form when $k \to 0$, and as in the case of the particle solutions, the two approach the same value when $k \to \infty$ as shown in equation (9):

$$\lim_{k\to\infty}\left|\Psi^{(3)}\right\rangle = \lim_{k\to\infty}\left|\Psi^{(4)}\right\rangle = \frac{1}{2}\sqrt{\frac{E+m}{2m}} \begin{bmatrix} \frac{p_1-ip_2+p_3}{(E+m)} \\ \frac{p_1+ip_2-p_3}{(E+m)} \\ 1 \\ 1 \end{bmatrix} e^{-i(Et-px)} \tag{9}$$

For positrons, we will denote the boson form as e_b^+.

3. Some Issues Regarding this Type of Transition

Fermions such as electrons that transform into bosons would represent a new form of matter. This brings up some issues that could be significant in connection with such transformations, and thus in connection with empirical observations. Below some topics are sketched that will require further elucidation beyond the scope of this paper.

a) Lepton number conservation. Leptons are usually characterized as elementary particles of spin 1/2 that are not subject to the strong force. It is not spin that defines leptons, however, but the fact that they are not subject to the strong force; hadrons are also spin ½ but *are* subject to the strong force. The two main classes of leptons are charged leptons, including the electron, muon, and tauon, and neutral leptons, i.e., neutrinos. Conservation of lepton number is a principle of quantum mechanics and quantum field theory (Schwartz, 2014), though violated in some grand unified theories (Klauber, 2024). There are two possibilities: (1) lepton number is not conserved under the conditions where fermions convert to bosons because we are dealing with a new state of matter; or (2) lepton number is conserved but the definition must be expanded to include particles of integer spin (transformed fermions) that are not subject to the strong force. That is, transformed fermions still retain their lepton identity, except that they have integer spin. This is the most likely explanation, since the important characteristic of leptons is their relationship to the four forces.

b) Energy release upon collapse. When White Dwarfs or Neutron stars collapse, a large amount of energy is released. There are two sources of potential energy involved (1). When electron degeneracy pressure fails, the Pauli Exclusion Principle (responsible for the size of atoms) no longer functions, and according to the theory advanced in the article, this means that electrons no longer need to inhabit orbits far from the nucleus, so large amounts of electrical potential energy would be freed (2). The physical collapse of the White Dwarf to the much smaller size of the Neutron star due to the collapse of atoms would release large amounts of gravitational potential energy.

c) Electron interaction with protons. With electrons no longer subject to the Pauli principle, it is not clear how they would interact with protons. Free neutron decay, $n^0 \to p^+ + e^- + \overline{\nu}_e$, which in some cases also involves a gamma ray (photon) emission as well, suggests that the reverse process whereby an electron and proton combine to form a neutron would not be favored energetically. It is also not clear whether or how such a reverse process could occur if the electron now has integer spin. If the electrons cannot combine with protons due to integer spin, there might be some type of electron "gas" or "flood" around protons, very roughly analogous to what happens in metals and some semiconductors, though without the energy level quantization.

d) Quantum statistics. These statistics (Bose-Einstein and Fermi-Dirac) are descriptive, not prescriptive, of the aggregate behavior of particles under quantum mechanics. That is, spin determines the statistics, not the other way around. Consequently, under extreme conditions, if particle spin can change, the statistics would automatically change as well. The fundamental nature of the connection between spin and statistics, and what happens if a particle transitions from half integer to integer spin, is an important subject.

e) Conservation of angular momentum. Obviously, any change of spin involves a change of angular momentum, since the angular momentum of a particle is given in units of spin and Planck's constant. Angular momentum is always conserved, so if spin goes from ½ to 0, there are two possibilities: (1) If there is significant imbalance in +1/2 and -1/2 spin particles, the excess could go to the angular momentum of the Neutron star or to flowing currents within the star. (2) More likely, the number of +1/2 and -1/2 spin particles is approximately the same, so the net change in angular momentum would be near zero, with any excess distributed among currents in the star or the star's angular momentum. It would be most valuable to measure the angular momentum of an electron in the new state, and its gyromagnetic ratio g_e, but given the requirements for transforming an electron, that may not be possible in earthbound laboratories.

4. Functional Form of Expression Governing Transition from Fermion to Boson

For this theory to be viable, it is necessary to find an expression for the exponent k which involves factors associated with the extreme conditions in White Dwarfs and Neutron Stars. These factors include mass, density, degeneracy pressure, and spacetime curvature. According to General Relativity, spacetime is curved by both mass and pressure. The contribution due to pressure—the degeneracy pressure—is much stronger than that due to mass. For spacetime curvature, it is reasonable to assume that some relationship involving the Riemann Curvature Tensor is involved. The resulting expression for $e^{1/k}$ must approximate a step function, so that the transition from fermion to boson occurs very rapidly as conditions approach those of a White Dwarf or Neutron Star.

Unfortunately we do not have a good understanding of the physics of White Dwarfs or Neutron Stars, which would allow us to determine expressions for the spacetime curvature.

The theory advanced here requires that k be a scalar function (rank 0 tensor), but the Riemann Curvature Tensor and the Ricci Tensor are both higher-order tensors. Since we do not know well the physics in the Neutron Star, we consider the Schwarzschild solution to Einstein's Field Equations. The Schwarzschild solution applies strictly in the case of a static, spherically symmetrical field near a massive spherical object. This is not exactly the description of the interior of a Neutron Star, but it is at least computationally tractable, and can be used in the case of simple Black Holes (Collier, 2014; Moore, 2013). If, as appears to be the case, that the Neutron Star is much denser toward the center, the Schwarzschild metric may be adequate as a first approximation to estimate spacetime curvature. We will use this assumption and see how far we can get with it.

The Schwarzschild metric tensor takes the form of equation (10):

$$g_{\mu\nu}(t,r,\theta,\phi)=\begin{bmatrix} 1-\dfrac{2GM}{c^2 r} & 0 & 0 & 0 \\ 0 & \dfrac{c^2 r}{2GM-c^2 r} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{bmatrix} \quad (10)$$

The Ricci curvature scaler R= 0 for this metric, so it cannot be used. However, the Tensor Norm (a rank 0 tensor) can be employed as a measure of the curvature. For any tensor, the norm is defined as the square root of the tensor contracted with itself in all indices (Shoshany, 2021). For the Schwarzschild metric, the Norm squared is defined and calculated as shown in equation (11):

$$NS^2=\left|g_{\mu\nu}\right|^2=g^{\mu\nu}g_{\mu\nu}=\frac{48G^2M^2}{c^4r^6} \quad (11)$$

The corresponding norm is given in equation (12):

$$NS=\frac{16\sqrt{3}\pi GM}{3Vc^2} \quad (12)$$

assuming a sphere so that the r term can be expressed in terms of volume V.

In this discussion, we will use standard mks units rather than natural units because the mks units give an immediate indication of the values of the forces involved. After much experimentation, the following formula looks to be fairly accurate for the constant that determines transition to boson behavior, equation (13):

$$k=1000\exp\left(-\frac{2.5\cdot 10^{49}}{V_0^2\cdot PD\cdot NS}\right) \quad (13)$$

where V_0= initial volume of star (before any collapse has started), PD = Degeneracy Pressure, and NS = Schwarzschild Norm. The last two are functions of the volume of the star as it is collapsing. This formula works for both collapse to Neutron Star from White Dwarf (electrons-> bosons) and collapse of Neutron Star to Black Hole (protons/neutrons-> bosons). In both cases, degeneracy pressure is overcome by gravity, i.e., spacetime curvature, as measured by the Schwarzschild Norm. The advantage of the formula is that it does not require separate constants for the different cases of White Dwarf and Neutron Star, and also that it utilizes both degeneracy pressure and a General Relativity measure of spacetime curvature.

Use of the Schwarzschild Norm requires assumptions about the solution to the Einstein Field Equations for the particular case. As noted, we do not have a good understanding of the physics of the stars involved, so as a first approximation we will assume that the Schwarzschild solution is applicable and use the Schwarzschild Norm to estimate spacetime curvature inside the collapsing star. The Neutron Star is rotating but is not yet a Black Hole, so we can for approximation purposes prescind from the Kerr metric.

The formula gives following graphs (Figure 1 and Figure 2):

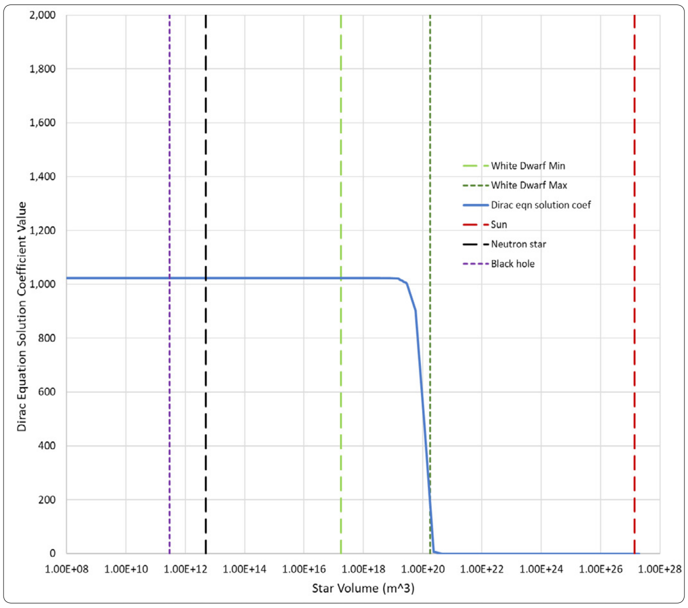

Figure 1. Electron Degeneracy Case (electrons->bosons)

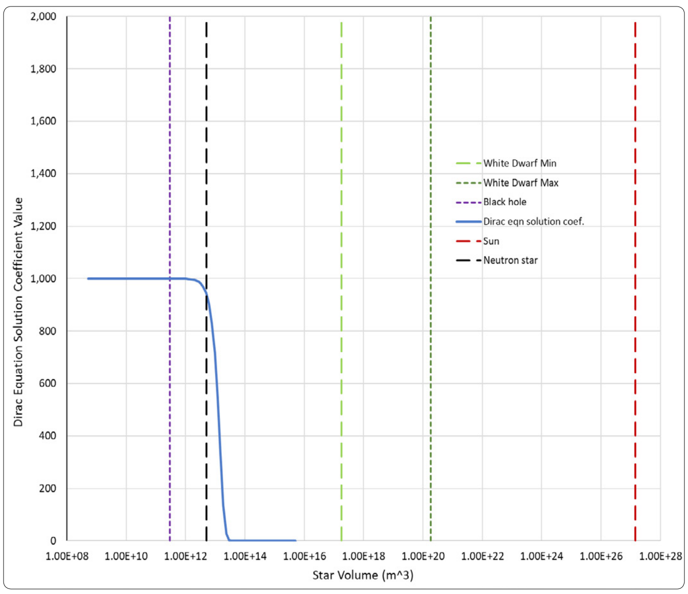

Figure 2. Proton/Neutron Degeneracy Case: Baryons-> Bosons

These graphs show that the same formula can explain at least to first order the very rapid collapse of White Dwarfs to Neutron Stars, and the collapse of Neutron Stars to Black Holes.

5. Application to Magnetic Field in Magnetars

In this section we consider an application of the theory that electrons can become bosons. The goal is not to create a new theory of Neutron star structure or a new equation of state (Moskvitch, 2020). Current theories postulate that under the extreme conditions of Neutron stars, charged particles can form a superfluid [refs], to generate the required magnetic fields. There are some problems with this theory, due to the fact that the temperatures inside a neutron star far exceed those usually associated with superfluidity as we currently understand it, which requires much lower temperature than that assumed for neutron stars, about 10^{11} for newly formed stars, and 10^{6} for older ones (Lattimer, 2015). The purpose here is not to solve the problem of the state of neutron star cores, and whether they are or contain superfluids, but to show that the conversion of fermions into bosons, resulting in a new state of matter, can achieve flows consistent with the observed large magnetic fields of magnetars, owing to the extremely high density of electrons now possible in such flows. Due to our lack of knowledge of the structure and physics of neutron stars, here we wish to show only that with some reasonable assumptions, the hypothesis of boson behavior of electrons can yield magnetic field strengths in the approximate range of those observed or inferred from observations.

The problem is how the extremely strong magnetic fields of Magnetars are generated. Obviously, Neutron Stars cannot be composed entirely of neutrons. The kind of magnetic field strengths observed, 10^{9} – 10^{11} T (McGill, 2020), require large flowing currents, some type of dynamo (Moskvitch, 2020). In the present case, we will utilize the hypothesis advanced here that the electrons become bosons, and see if such particles can explain the magnetic field. We can assume that the modified electron is still subject to the same forces as before: gravity, E&M, weak. One question is whether all of these electrons will combine with protons to form neutrons, or whether they will remain separate but still interact. Likely the latter since some sort of circulating current must be responsible for the extremely strong magnetic fields in neutron stars. Differences in field strength could be the result of different numbers of electrons not combining with protons. But to get circulating currents, it must be the case that some electrons (and possibly protons) form a fluid, possibly with electrons rotating one way and protons another. Here we shall consider electrons only. The electrons can now easily form a fluid since they are bosons, but the protons cannot—they are still subject to the Pauli Principle. The question, therefore, is how such fluids could work, since the electrons are mutually repulsive, as are the protons. The answer seems to be that the extreme pressure inside a Neutron Star can force electrons close together because it can overcome the repulsive electrical force, and there is no Pauli principle forcing them into different energy levels. In this way, extremely high densities of electrons can be achieved. Theory predicts that collapse would radiate from the center, where spacetime curvature (dependent on both mass and pressure) is maximum, out to the edges, as they are pulled in.

It is possible to estimate the magnetic field of a magnetar by making a few assumptions, and using the fact that electrons, now as bosons, can be put closely together without energy level issues stemming from the Pauli Exclusion Principle. We use the following, in addition to standard constant values:

- M_n = 1.68 x 10^{-27} kg (mass of neutron)
- R_e = 2.80 x 10^{-15} m (radius of electron)
- μ_0 = 1.2664 x 10^{-6} N/A^2 (assumed permittivity)
- R_{ns} = 1.00 x 10^{4} m (radius of Neutron Star)
- M_{NS} = 8.36 x 10^{24} kg (mass of Neutron Star, taken as 1.4 x mass of Sun)
- V_{NS} = 4.19 x 10^{12} m^3 (volume of Neutron Star)
- B = 1.00 x 10^{11} T (mag field of Neutron Star)
- E_{cs} = 2.50 x 10^{-29} m^2 (cross section of electron)

We shall assume that the magnetic field is generated by electrons rotating around some central axis passing through the core, and that the axis may or may not be aligned with the axis of rotation. Initially it is assumed to be fairly close. This will be modelled as a solenoid for purposes of calculating the magnetic field, with "virtual wires" of electrons spaced closely (electron centers approximately 1 electron radius apart). While electrons are considered to be point particles, they are surrounded by virtual particles, so they have an approximate finite radius. The electrons are kept apart by electrostatic repulsion only, not by any effect of the Pauli Exclusion Principle, since they are now acting as bosons. That repulsive force is given by Coulomb's Law, equation (14):

$$F_e = \frac{kq^2}{r^2} \tag{14}$$

For two electrons with centers separated by 2x electron radius*r*, the value of this force is calculated in equation (15):

$$F_e = \frac{kq^2}{r^2} = \frac{8.99 \cdot 10^9 \cdot (1.6 \cdot 10^{-19})^2}{(2.80 \cdot 10^{-15})^2} = 29.4N \tag{15}$$

To see that gravity cannot overcome this force, we can calculate the gravitational force on an electron at the surface of a Neutron Star, given as equation (16):

$$F_g = \frac{G \cdot M_{NS} M_e}{R_{NS}^2} = \frac{6.64 \cdot 10^{-11} \cdot 2.80 \cdot 10^{30} \cdot 9.11\text{x } 10^{-31}}{\left(1.00 \cdot 10^{4}\right)^2} = 1.69 \cdot 10^{-18} N \tag{16}$$

In a Neutron Star, the star is held up only through the degeneracy pressure of neutrons, which are still acting like fermions; electrons have been overwhelmed and are acting like bosons. We will assume that the degeneracy pressure can be estimated using the formula for Fermionic fluids. The degeneracy pressure that acts on particles, including electrons, in the context of stars, is given by equation (17) [Branson, (2013)]:

$$P_D = \frac{\pi^3\hbar^2}{15m}\left(\frac{3N}{\pi}\right)^{5/3} V^{-5/3} = \frac{(3\pi^2)^{2/3}\cdot\hbar^2\cdot(N/V)^{5/3}}{5M_e} \quad (17)$$

where N/V = number of atoms/unit volume, and M_e is mass of the electron. In this case, N is calculated from the mass of the Neutron Star, and V from the radius. Note that this formula is different than that in #417, which is probably in error since this formula has the correct units. The number of particles is calculated as equation (18):

$$N = \frac{M_{NS}}{M_n} = \frac{2.80\cdot10^{30}}{1.68\cdot10^{-27}} = 1.67\cdot10^{57} \quad (18)$$

Then N/V is calculated using the volume of the Neutron Star in equation (19):

$$N/V = \frac{1.67\cdot10^{57}}{V_{NS}} = \frac{1.67\cdot10^{57}}{4.19\cdot10^{12}} = 3.99\cdot10^{44} \text{ particles/m}^3 \quad (19)$$

The degeneracy pressure inside the Neutron Star can then be calculated as equation (20):

$$\begin{aligned} P_D &= \frac{(3\pi^2)^{2/3}\cdot\hbar^2\cdot(N/V)^{5/3}}{5M_e} \\ &= \frac{(3\pi^2)^{2/3}\cdot(1.0546\cdot10^{-34})^2\cdot(3.99\cdot10^{44})^{5/3}}{5\cdot9.11\cdot10^{-31}} \text{ N/m}^2 \\ &= 5.05\cdot10^{36} \end{aligned} \quad (20)$$

Units check with this calculation—kg/(m s^2) = Pa = N/m^2. The force on an electron would then be given by equation (21):

$$F_{e.P_D} = P_D\cdot E_{CS} = 5.05\cdot10^{36}\cdot2.5\cdot10^{-29} = 1.26\cdot10^{8} \text{ N} \quad (21)$$

This is far greater than the repulsive electrostatic force between two electrons. To estimate the magnetic field strength, we will assume a solenoid-type of current, with electrons flowing through "virtual wires" around a core consisting of protons. Given the radius of the Neutron Star, and assuming that the virtual wires of the solenoid are spaced at about 1 electron diameter apart, over the length of the solenoid = diameter of the Neutron Star, we would have, for the number n of turns, equation (22):

$$n = \frac{2\cdot R_{NS}}{2\cdot R_e} = \frac{2\cdot10^4}{2\cdot2.8\cdot10^{-15}} = 3.57\cdot10^{18} \quad (22)$$

We assume that the electrons are densely packed. Assuming that we want a magnetic field of 10^{11} T, we can estimate the solenoid current as equation (23):

$$A = \frac{B}{\mu_0\cdot n} = \frac{10^{11}}{1.2664\cdot10^{-6}\cdot3.57\cdot10^{18}} = 2.21\cdot10^{-2} \; amps \quad (23)$$

Electrons per second N_e needed to generate this current in the solenoid, with the given number of "turns", is calculated as equation (24):

$$N_e = \frac{A}{q} = \frac{2.21\cdot10^{-2}}{1.60\cdot10^{-19}} = 1.39\cdot10^{17} \quad (24)$$

The average velocity of the electrons needed to generate the current V_e is equation (25):

$$V_e = N_e\cdot2\cdot R_e = 1.39\cdot10^{17}\cdot2.8\cdot10^{-15} = 779 \text{ m/s} \quad (25)$$

This is comfortably less than the speed of light. We do not know the radius of the solenoid, but we can construct a table that relates the radius of the solenoid to its rotation frequency (assumed to be the rotation frequency of the Neutron Star), and the number of electrons/virtual wire and the total number of electrons (Table 1):

radius of solenoid (m)	f (rot/sec)	# electrons/wire	Total # electrons
25	4.959	5.61E+16	2.00E+35
50	2.479	1.12E+17	4.01E+35
100	1.240	2.24E+17	8.01E+35
250	0.496	5.61E+17	2.00E+36
500	0.248	1.12E+18	4.01E+36
1000	0.124	2.24E+18	8.01E+36

Table 1. Solenoid calculations for hypothetical model of neutron star magnetic field strength

Since the rotation speed of Magnetars is known to be 1 to 10 times per second, this would, according to the above table, correspond to a solenoid radius of about 100-1000 m.

It is well-known that the rotation axis of a Magnetar need not be the same as the axis of its magnetic field. This explains the "lighthouse effect" that allows us to "see" these objects. If we let the angle between the rotation axis and the magnetic field be θ, the rotation of the star would still cause rotation of the electrons in the solenoid, at the same frequency, under the assumption that the rotation of the electrons is coupled to the rotation of the star. It can be shown that the effective radius r_{eff} of the solenoid decreases as cosθ, becoming zero when the axes are orthogonal (q=90°). Specifically, $r_{eff} = r_{sol}$ cosθ, where r_{sol} is the normal radius of the solenoid. If the rotation frequency is the same, and the number of electrons is the same, but the effective radius decreases, then the velocity of the electrons must decrease, which corresponds to a decrease in the current and thus in the magnetic field strength. If r_{sol} is the radius of the virtual solenoid, and f is the rotation frequency of the star, it can be shown that magnetic field strength is given by equation (26):

$$B = \frac{2\pi r_{eff} f q\mu_0 n}{R_e} = \frac{2\pi r_{sol}\cos\theta\, f q\mu_0 n}{R_e} \quad (26)$$

When they are orthogonal, the magnetic field strength would decrease to 0. However, if the electron movement is not due to the star's rotation, this would not necessarily be the case.

6. Electrons as Bosons: The Long-sought "Superpartners" of Supersymmetry theory?

Supersymmetry theory postulates superpartners of ordinary particles as a means to solve some outstanding problems of the Standard Model of particle physics (Murayama, 2000a). In particular,

For every particle, there is a superpartner whose spin differs by 1/2. By doubling the number of particles again, there is similar cancellation between the process with ordinary particles only and another process with their superpartners. Then the Standard Model can describe physics down to the Planck length [10^{-33} cm], making the marriage a realistic hope. In fact, it is a necessary ingredient in the only available candidate for quantum theory of gravity, string theory (Murayama, 2000b).

An admittedly speculative application of the theory that fermions can become bosons under extreme conditions can be made in connection with superpartners of Supersymmetry theory. These "superpartners", such as that of the electron, the "selectron", have never been found, despite decades of search, including experiments with the Large Hadron Collider at CERN. This would be the case if the selectron is an electron that can only transition to its boson superpartner under the extreme conditions in a White Dwarf poised to collapse, which cannot be created in earthbound laboratories. As noted, the requirement for a superpartner is for it to have a spin that is ½ unit different than its partner. In the case of fermions such as electrons and neutrons, a spin of 0 would suffice. This number naturally emerges from the modified Dirac equation solutions. A further possibility is that the symmetric case also obtains: conversion of bosons to fermions under the reverse conditions, namely exceedingly low value of spacetime curvature and temperature. Such conditions would only exist in realms of space far removed from baryonic matter and dark matter, and likewise could not be simulated in any earthbound laboratory. This does not solve any outstanding problems in supersymmetry theory, only suggests a reason why superpartners have not been found.

7. Conclusions and Future Work

Failure of electron or neutron degeneracy pressure causing collapse of White Dwarfs and Neutron Stars is partially explained by showing that under suitable conditions, fermions can become bosons. This does not require a new theory of fermions, but only a modified solution to the Dirac equations. The theory can be applied to explain the magnetic field strength of magnetars. Future work on this topic includes:

- How lepton number conservation behaves under transformation conditions.
- Calculation (estimation) of energy release due to atomic collapse and subsequent stellar collapse.
- Nature of transformed electron interaction with protons.
- Connection between spin and statistics under transformation conditions.
- Measurement of angular momentum and gyromagnetic ratio of transformed electrons.
- Research into ways of detecting evidence for conversion of bosons into fermions in empty realms of space.

References

1. Branson James. Quantum Physics. 130; University of San Diego.
2. Collier Peter; A Most Incomprehensible Thing Notes Toward a Gentle Introduction to the Mathematics of Relativity. 2nd edition, Incomprehensible Books. 2014; 238-240.
3. Klauber Robert. Student Friendly Quantum Field Theory. Sandtrove Press. 2013; 88.
4. Klauber Robert. private correspondence. 2024.
5. Lattimer James M. Introduction to neutron stars. *American Institute of Physics Conference Series. AIP Conference Proceedings*. 2015; 1645(1): 61–78.
6. Lequeux James. Birth, Evolution, and Death of Stars. World Scientific. 2013; 90. doi: 10.1142/8820
7. McGill University. McGill Online Magnetar Catalog.
8. Moore Thomas. A General Relativity Workbook. University Science Books. 2013; 106.
9. Moskvitch Katia. Neutron Stars. Harvard University Press. 2020; Pages: 110-111, 131-162, 131-162.
10. Murayama Hitoshi. Introduction to Supersymmetry. 2000b.
11. Murayama Hitoshi. Supersymmetry Phenomenology. 2000a. 13.
12. Schwartz Matthew. Quantum Field Theory and the Standard Model. Cambridge University Press. 2014; 635-636.
13. Schwichtenberg Jacob. No-Nonsense Quantum Field Theory. No-Nonsense Books. 2020. 198.
14. Shoshany Barak. OGRe: An Object-Oriented General Relativity Package for Mathematica. Documentation. 2021; 75-76.