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The Impact of Magnetic Dipole Radiation and Decay on the Variation of Period and Inclination Angle of Pulsars

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Abstract

The change of pulsar period and inclination angle under the braking of magnetic radiation and magnetic decay is studied. The system of equations of evolution of period and inclination angle is given and solves it. The solution shows that the period increases and inclination angle decreases with time under the braking of magnetic radiation and decay. The numerical results for the change of period and inclination angle of PSR0531+21 are given. The discussions are drawn.

Keywords: Pulsar; Period and inclination angle; Braking of magnetic radiation and Decay; Change.

Introduction

When pulsar was born, the gravitational radiation plays a leading role in 81 year [1]. Afterward, the magnetic radiation plays a second role in a life of pulsar. In this stage pulsar rotation is very fast and its rotational energy is very large and its magnetic field is very stronger, but the rotational energy and the magnetic field also are small and weakness with time gradually. It is well known that the rotational energy may transforms to pulsar radiation energy and the weakness of magnetic field may result in the magnetic moment decay. Other hand the magnetic radiation and magnetic decay result in prolong of period and decrease of obliquity angle with time.

In the past years some authors researched these topics, such as, Ostriker and Gunn [1], Davns & Goltstein [2], Wang et al. [3], Heansel et al. [4], Mira et al. [5], Urpin & Gil [6], Philippov et al. [7]. The research of these authors is very desirable and devisable. Based on these researches, the present paper combines concrete pulsar for researching the evolution of the period and obliquity angle under braking of magnetic radiation and magnetic decay.

The Change of Pulsar Period and Inclination Angle under the Braking of Magnetic Dipolar Radiation

Philippov et al. provided a method for studding the evolution of period and obliquity angle of pulsar under braking of magnetic radiation and decay. This paper cited the system of equations with angular velocity Ω and inclination angle α [7].

$$I\frac{d\Omega}{dt} = K_Z \qquad (1)$$

$$I\Omega\frac{d\alpha}{dt}=K_x \tag{2}$$

where I dettes the moment of inertia. The Z-compo net of toque, K_z, act opposite to Ω, K_x component act perpendicular to $\vec{\Omega}$.

According to Philippov et al. [7]

$$K_x=\frac{2}{3}K_a\sin\alpha\cos\alpha \tag{3}$$

$$K_z=-\frac{2}{3}K_a\sin^2\alpha \tag{4}$$

where $K_a=\frac{\mu^2\Omega^3}{c^3}$ (5)

μ is magnetic moment

Substitution of (3), (4) and (5) into (1) and (2), we obtain

$$I\frac{d\Omega}{dt}=-\frac{2\mu^2\Omega^3}{3c^3}\sin^2\alpha \tag{6}$$

$$I\Omega\frac{d\alpha}{dt}=-\frac{2\mu^2\Omega^3}{3c^3}\sin\alpha\cos\alpha \tag{7}$$

where $\mu^2=(BR^3)^2=B^2R^6$ (8)

By letting $\Omega=2\pi/P$. P is period of pulsar.

The equations (6) and (7) become as

$$\frac{dP^2}{dt}=\frac{16\pi^2\mu^2}{3c^3I}\sin^2\alpha \tag{9}$$

$$\frac{da}{dt}=-\frac{8\pi^2\mu^2}{3c^3IP^2}\cos\alpha\sin\alpha \tag{10}$$

The equations (9)-(10) can be given

$$\frac{dP^2}{P^2}=-\frac{2\sin\alpha}{\cos\alpha}d\alpha \tag{11}$$

Integrating the above equation

$$P=P_0\left(\frac{\cos\alpha}{\cos\alpha_0}\right)\quad \Omega=\Omega_0\left(\frac{\cos\alpha_0}{\cos\alpha}\right) \tag{12}$$

The equation (10) can be written by using (12):

$$d\alpha=-\frac{8\pi^2\mu^2}{3c^3IP_0^2}\cos^2\alpha_0\left(\frac{\sin\alpha}{\cos\alpha}\right)dt,$$

$$\therefore\frac{d\alpha}{\tan\alpha}=-\frac{8\pi^2\mu^2}{3c^3IP_0^2}\cos^2\alpha_0 dt$$

Integrating the above equation

$$\int_{\alpha_0}^{\alpha}ctg\alpha d\alpha=-\frac{8\pi^2\mu^2}{3c^3IP_0^2}\cos^2\alpha_0\int_{t_0}^{t}dt \tag{13}$$

$$\mathrm{h}\left(\frac{\sin\alpha}{\sin\alpha_0}\right)=-\frac{8\pi^2\mu^2\cos^2\alpha_0}{3c^3P_0^2}(t-t_0).$$

$$\sin\alpha=\sin\alpha_0\exp\left[-\frac{8\pi^2\mu^2\cos^2\alpha_0}{3c^3IP_0^2}(t-t\ )\right]\quad \alpha=\arcsin\alpha \tag{14}$$

$$\cos\alpha=(1-\sin^2\alpha)^{1/2}=\left\{1-\sin^2\alpha_0\exp\left[-\frac{16\pi^2\mu^2\cos^2\alpha_0}{3c^3IP_0^2}(t-t_0)\right]\right\}^{1/2}.$$

Substituting the above expression into the first expression of (12), we obtain the change of period of pulsar

$$P(t)=P_0\sec\alpha_0\left\{1-\sin^2\alpha_0\exp\left[\frac{16\pi^2\mu_0^2\cos^2\alpha_0}{3c^3IP_0^2}(t-t_0)\right]\right\}^{1/2} \tag{15}$$

The expressions (14) and (15) are that the change of period and inclination angle with time under the braking of the magnetic radiation.

The Change of the Period and Inclination Angle under the Braking of the Magnetic Decay

We assume that the magnetic moment of pulsar decays with the exponent, i. e

$$\mu^2=\mu_0^2e^{-\xi t} \tag{16}$$

ξ is the coefficient of the magnetic decay. $\mu_0=R^3B_0$. Substituting (16) into the equation (13) and integrating it

$$\int_{\alpha_0}^{\alpha}ctg\alpha d\alpha=-\frac{8\pi^2\mu_0^2}{3c^3IP_0^2}\cos^2\alpha_0\int_{t_0}^{t}e^{-\xi t}dt$$

$$\ln\left(\frac{\sin\alpha}{\sin\alpha_0}\right)=-\frac{8\pi^2\mu_0^2\cos^2\alpha_0}{3c^3IP_0^2}\left[1-e^{-\xi(t-t_0)}\right],$$

$$\sin\alpha=\sin\alpha_0\exp\left\{-\frac{8\pi^2\mu_0^2\cos^2\alpha_0}{3c^3IP_0^2}\left[1-e^{-\xi(t-t_0)}\right]\right\}\quad \alpha=\arcsin\alpha \tag{17}$$

$$\cos\alpha=(1-\sin^2\alpha)^{1/2}\left\{1-\sin^2\alpha_0\exp\left[-\frac{16\pi^2\mu_0^2\cos^2\alpha_0}{3c^3IP_0^2}\left(1-e^{-\xi(t-t_0)}\right)\right]\right\}^{1/2}$$

Substituting the above expression into the first expression of (12), we obtain

$$P(t)=P_0\sec\alpha_0\left\{1-\sin^2\alpha_0\exp\left[-\frac{16\pi^2\mu_0^2\cos^2\alpha_0}{3c^3IP_0^2\xi}\left(1-e^{-\xi(t-t_0)}\right)\right]\right\}^{1/2} \tag{18}$$

The expressions of (17) and (18) are that the change of period and obliquity angle with time under their braking of the magnetic decay.

Numerical Results

This paper uses the formulae (14)–(15) and (17)–(18) to calculate the change of the period and magnetic inclination angle under the braking of magnetic radiation and magnetic decay for PSR0531+21 (Crab pulsar) [8]. Its period P_0=0.033058(*s*), and magnetic field $B_0=3.78 \times 10^{12} G$ cited from Manchestefer et al. [9]. It's obliquity angle α_0=59°.2 cited from Davis and Goldstein [2]. $I=1.4 \times 10^{45}$ ($g.cm^2$) [8], $R=1.2 \times 10^6$ *cm*. [3], $\xi=1.1111 \times 10^{-6}/yr$ [3].

$$P(t) = P_0 \sec\alpha_0 \left\{1 - \sin^2\alpha_0 \exp\left[-\frac{16\pi^2 \mu_0^2 \cos^2\alpha_0}{3c^3 I P_0^2 \xi}\left(1 - e^{-\xi(t-t_0)}\right)\right]\right\}^{1/2}$$

Substituting these data into the formulae (14)-(15) and (17)-(18), the numerical results are listed in tables 1 and 2.

Table 1. The change of the period and inclination angle under the braking of the Magnetic dipolar radiation.

Pulsar	$P_0(s)$	$P(t)\left(s/cy\right)$	$\delta P\left(s/cy\right)$	α_0(deg)	$\dot{\alpha}(t)\left(\mathrm{deg}/cy\right)$	$\delta\alpha\left(\mathrm{deg}/cy\right)$
PSR0531+21	0.033085	0.0368690	0.0037840	59.2	57.1269	-2.0731

Table 2. The change of the period and inclination angle under the braking of the magnetic decay.

Pulsar	$P_0(s)$	$P(t)\left(s/cy\right)$	$\delta P\left(s/cy\right)$	α_0(deg)	$\dot{\alpha}(t)\left(\mathrm{deg}/cy\right)$	$\delta\alpha\left(\mathrm{deg}/cy\right)$
PSR0531+21	0.033085	0.0368695	0.0037845	59.2	57.1266	-2.0734

Discussion and Conclusion for Numerical Results

(1) It can be seen from tables 1 and 2 that the period increases and inclination angle decreases with time under the magnetic radiation and magnetic decay.

(2) However the increase of period and decrease of inclination angle under the braking of magnetic decay are larger than that under the braking of magnetic radiation.

(3) The decrease of magnetic radiation is due to the rotational energy loss and the magnetic decay is due to the weakness of magnetic field with time. Hence the increase of period and decrease of the magnetic inclination angle are due to the rotational energy loss and the weakness of the magnetic field of pulsar.

(4) The pulsar PSR0531+21 speeds up suddenly due to stellar quake (glitches) in three years [10]. However it is temporary happening and it is not secular happening. It does not influence the secular variation of period and inclination angle due to the braking of magnetic radiation and magnetic decay.

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