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Kaluza-Klein FRW type Perfect Fluid Cosmological Models with Linearly varying Deceleration Parameter in a Modified Gravity

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Abstract

Here, we investigate cosmological models with perfect fluid source in the framework of Kaluza- Klein FRW space time in a modified theory of gravity known as *f (R, T)* gravity (Harko et al. 2011) where *R* is the Ricci scalar and *T* is the trace of the stress energy tensor of the perfect fluid. By solving the field equations, we have found an exact solution of the field equations using a linearly varying deceleration parameter which can be considered as a five dimensional FRW type cosmological model in *f (R, T)* gravity. We have determined the spatial volume and average Hubble parameter of the model. We have also determined and discussed the physical parameters, pressure and density for dust, radiation and stiff matter dominated eras.

Keywords: Kaluza-Klein model; FRW models; Cosmological models; Perfect fluid; f (R, T) gravity.

Introduction

It s well known that Einstein's theory of gravity has been successful in describing the gravitational phenomena and cosmology and cosmological models of the universe. However, it is said that this theory does not fully account for certain aspects of present day cosmology. To mention some of them, we can say that Einstein's theory does not fully incorporate famous Mach's principle, it does not avoid singularity problem and does not explain the modern scenario of accelerated expansion of the universe. Riess et al. [1] and Perlmutter et al. [2] have confirmed that the universe is not only expanding but it is accelerating by analyzing their Supernova 1a experimental data. It is also proposed that a huge negative pressure known as dark energy is responsible for this cosmic acceleration. In order to explain this two approaches have been proposed. One way is to investigate several dark energy candidates [3-7] and another way is to modify Einstein's theory of gravitation. Hence modifying Hilbert-Einstein action some modified theories of gravitation have been formulated. Significant among them are *f (R)* gravity [8], *f (R, T)* gravity [9] and scalar-tensor theories of gravitation constructed by Brans-Dicke [10] and Saez and Ballester [11]. Here we are interested in *f (R, T)* gravity.

A modified theory of gravity known as *f (R, T)* gravity has been formulated by Harko et al. [9], using the action

$$S=\frac{1}{16\pi}\int f(R,T)\sqrt{-g}\,d^4x+\int L_m\sqrt{-g}\,d^4x, \tag{1}$$

have obtained the gravitational field equations of the theory. Here the gravitational Lagrangian L_m is given by an arbitrary function of Ricci scalar *R* and of the trace *T* of the energy momentum tensor T_{ij}. Generally, the gravitational field equations depend on nature of the matter source. The gravitational field equations of *f (R, T)* gravity are obtained by varying the action given by Eq. (1) with respect to the metric g_{ij} (the detailed derivation is found in Harko et al. [9]) as

$$f_1'(R)R_{ij}-\frac{1}{2}f_1(R)g_{ij}=8\pi T_{ij}+f_2'(T)T_{ij}+\left[f_2'(T)p+\frac{1}{2}f_2(T)\right]g_{ij},\quad (2)$$

Here prime indicates the derivative with respect to the argument, the function *f (R, T)* is taken as

$$f(R,T) = R + 2f(T) \quad (3)$$

where *f (T)* is an arbitrary function of the trace of the stress energy tensor given by

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \quad (4)$$

ρ and p being the density and isotropic pressure, respectively of the fluid, u_i is the four velocity of the comic fluid and the other symbols in Eq. (2) have their usual meaning. Also, Harko et al. [9] presented the field equations of several particular models corresponding to some other explicit forms of the function *f (R, T)* .

Several interesting cosmological models have been investigated in *f (R, T)* theory of gravitation by choosing appropriate function *f (T)* . Harko et al. [9] discussed FRW cosmological models. They have also discussed the case of scalar fields and Brans-Dicke (BD) type formulation of the model which play an important role in cosmology. Many authors have investigated perfect fluid cosmological models in this particular theory. Adhav [12] discussed Bianchi type-*I* perfect fluid model in *f (R, T)* gravity while Chandel and Shri Ram [13] and Reddy et al. [14] have investigated several aspects of Bianchi type-*III* perfect fluid cosmological models in this theory. Sharif and Zubair [15] have obtained anisotropic universe models with perfect fluid and scalar field in this particular theory of gravity. Rao and Neelima [16] discussed perfect fluid Einstein-Rosen universe in this theory while Kaluza-Klein perfect fluid models in this theory are given by Reddy et al. [17]. Mishra et al. [18] have studied non-static cosmological model in this theory of gravity. Aditya et al. [19] obtained Bianchi type-*I*, *VIII* and *IX* perfect fluid models in *f (R, T)* gravity. Also, Rao et al. [20] have considered Bianchi type-VI_h perfect fluid model in this theory. Santhi et al. [21] have investigated Kantowski-Sachs scalar field cosmological models in *f (R, T)* gravity. Ramesh and Umadevi [22] have discussed FRW cosmological model in the presence of perfect fluid in this theory. Higher dimensional spherically symmetric models of the universe filled with perfect fluid source in *f (R, T)* gravity have been studied by Samanta and Dhal [23]. The theory of Kaluza-Klein (KK) [24, 25] is the result of an attempt to formulate five dimensional general relativity in which extra dimensions is used to couple the gravity and electromagnetism. An excellent review of this theory has been presented by many authors [26-28]. Darabi [29] in an interesting paper, tudied the non-compact, non-Ricci KK theory and has shown that the field equations with suitable equation of state (EoS) describe the early inflation and late time acceleration. Santhi et al. [30, 31] have studied KK cosmological models with two fluids source in Brans-Dicke and Saez-Ballester scalar-tensor theories of gravitation. Jamil and Debnath [32], Pradhan et al. [33], Ozel et al. [34] and Sharif and Farida [35] discussed FRW type KK cosmology in general relativity.

The present work is motivated by the above discussion. Here we consider non-Ricci, non-compact FRW type KK cosmological model in the presence of perfect fluid source in the framework of *f (R, T)* gravity. We organize the paper as follows: in Sect. 2, the field equations and the KK model are presented.Sect. 3, is devoted to the discussion of the behavior of cosmological parameters of the model. Summary and conclusions are given in Sect. 4.

f (R, T) gravity field equations and KK models

The geometry of FRW type KK cosmology is defined by the following metric [33, 34]

$$ds^2 = dt^2 - a^2\{\frac{dr^2}{1-kr^2} + r^2(d\xi^2 + sin^2\xi d\phi^2) + (1-kr^2)d\psi^2\} \quad (5)$$

where *a(t)* is the scale factor of the universe, *k*= -1, 0, +1 is the curvature parameter for open, flat, closed models respectively. We assume the 5D energy momentum tensor for perfect fluid as

$$T_i^j = (\rho + p)u_i u^j - \delta_i^j p \quad (6)$$

here the five velocity vector u^i satisfies $u_i u^i = 1$.

Now, using comoving coordinates and [9]

$$f(T) = \lambda T,\ \lambda = constant \quad (7)$$

we can write down the field equations of *f (R, T)* gravity given by Eq. (2) with the help of Eq. (6) for the metric (5) as

$$3\frac{\ddot{a}}{a}+3\frac{\dot{a}^2}{a^2}+3\frac{k}{a^2} = -p(8\pi+4\lambda)+\rho\lambda \quad (8)$$

$$6\frac{\dot{a}^2}{a^2}+6\frac{k}{a^2} = \rho(8\pi+3\lambda)-2p\lambda \quad (9)$$

where an overhead dot denotes differentiation with respect to cosmic time *t*. The above system consists of two equations but three unknowns (*a(t)*, ρ and *p*), hence is not fully determined. One may determine the system fully by specifying a theory that determines a relation between the energy density and pressure of the fluid. Most of the perfect fluids relevant to cosmology obey an equation of state of the form

$$p = \omega\rho \quad (10)$$

where ω is the equation of state (EoS) parameter, not necessarily constant. It may be noted that when ω is constant we obtain the well known cosmological fluid models-dust model (ω=0), stiff fluid model (ω=1), radiating model ($\omega=\frac{1}{3}$) and false vacuum or vacuum energy model (ω = –1). In literature, it is well known, that several authors have obtained perfect fluid models by solving the field equations of general relativity and modified theories of gravitation using the law of variation for Hubble's parameter proposed by Berman [36] and Berman and Gomide [37] which yields constant deceleration parameter models

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = m-1 \quad (11)$$

where $m \geq 0$ is a constant. Here we solve the field equations (8) and (9) using linearly varying deceleration parameter (LVDP) proposed by Akarsu and Dereli [38] given by

$$q = -kt + m - 1 \tag{12}$$

where $k \geq 0$ and $m \geq 0$ are constants and $k = 0$ reduces Eq. (12) to the Hubble's law of Berman [36] which yields constant deceleration parameter models of the universe. Ramesh and Umadevi [22] have investigated FRW cosmological models with LVDP in *f (R, T)* gravity. Sarkar [39] have obtained Bianchi type-V holographic dark energy universe with LVDP. Sahoo and Sivakumar [40] have studied LRS Bianchi type-I cosmological model in *f (R, T)* theory of gravity using this LVDP. Santhi et al. [41] have discussed Bianchi type-III magnetized holographic Ricci dark energy models using LVDP.

Following the discussion presented in [38], we observe that Eq. (12) leads to super exponential expansion unless $k = 0$. Now, solving Eq. (12) we obtain three different solutions given as

$$a(t) = a_0 \, exp\left(\frac{2}{\sqrt{m^2 - 2c_1 k}} arctanh\left(\frac{kt - m}{\sqrt{m^2 - 2c_1 k}}\right)\right) \quad k > 0, m \geq 0$$
$$a(t) = a_1 (mt + c_2)^{\frac{1}{m}}, \quad k = 0, m > 0 \tag{13}$$
$$a(t) = a_2 \, exp(c_3 \, t), \quad k = 0, m = 0$$

where a_0, a_1, a_2, c_1, c_2, c_3 are constants of integration. It may be noted that the last two of the solutions in Eq. (13) give us the constant deceleration parameter of the models of the universe.

Here we are interested in the perfect fluid model with linearly varying deceleration parameter model which is new. Now, choosing the integration constant $c_1 = 0$ and $m > 0$ we obtain

$$a(t) = a_0 \, exp\left(\frac{2}{m} arctanh\left(\frac{kt}{m} - 1\right)\right) \tag{14}$$

Now, using Eq. (14) in Eq. (5) the FRW type KK model in *f (R, T)* gravity can be written as

$$ds^2 = dt^2 - a_0^2 \, exp\left(\frac{4}{m} arctanh\left(\frac{kt}{m} - 1\right)\right)\left\{\frac{dr^2}{1 - kr^2} + r^2(d\xi^2 + sin^2 \xi \, d\phi^2) + (1 - kr^2) d\psi^2\right\} \tag{15}$$

Physical discussion of cosmological parameters

In this section,we determine the cosmological parameters of the model given by Eq. (15) and discuss their physical behavior in cosmology.

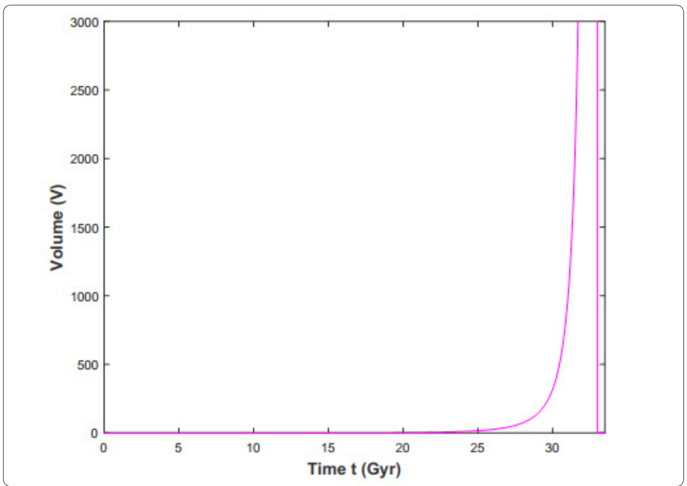

Figure 1. Plot of volume versus cosmic time t for a_0=1, k=0.097 and m=1.6.

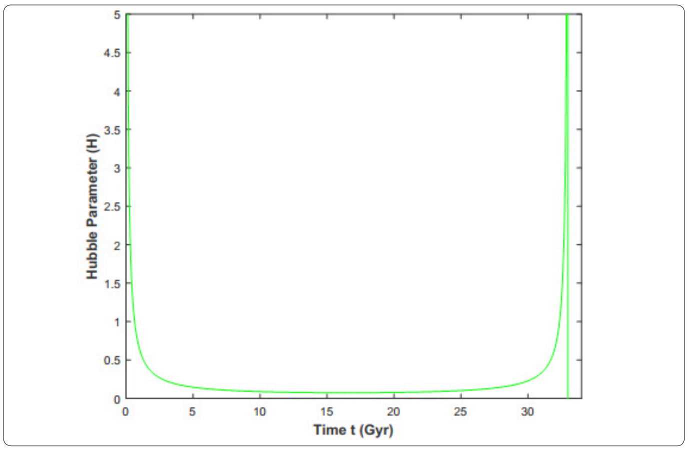

Figure 2. Plot of Hubble parameter versus cosmic time t for k=0.097 and m=1.6.

The spatial volume of the model is given by

$$V = a^4(t) = a_0^4 \, exp\left(\frac{8}{m} arctanh\left(\frac{kt}{m} - 1\right)\right) \tag{16}$$

The average Hubble parameter of the model is

$$H = \frac{\dot{a}}{a} = \frac{-2}{t(kt - 2m)} \tag{17}$$

Now, since the field equations (8) and (9) are highly non-linear we evaluate the physical parameters pressure and density in the universe (15) for dust, stiff matter and radiation dominated eras by substituting $\omega = 0, 1, \frac{1}{4}$ in Eq. (10).

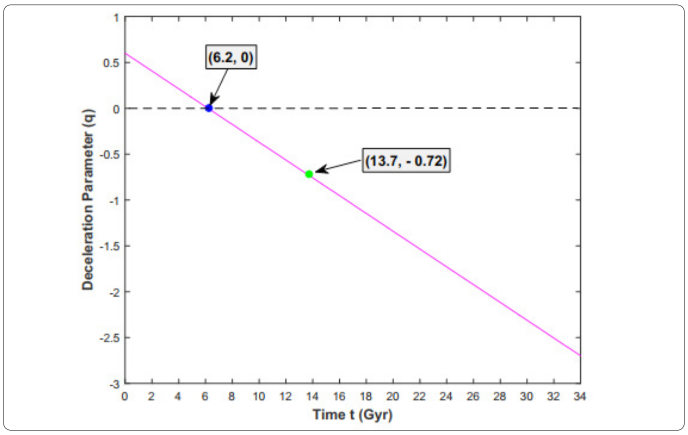

Figure 3. Plot of deceleration parameter versus cosmic time t for k=0.097 and m=1.6.

Case-1 (ω=0)

In this case we have from the field equations (8) and (9) the pressure and density as

$$p = 0$$
$$(8\pi + 4\lambda)\rho = \frac{12(4 - kt + m)}{t^2(kt - 2m)^2} + \frac{9k}{a_0^2} exp\left(\frac{-4}{m} arctanh\left(\frac{kt}{m} - 1\right)\right) \tag{18}$$

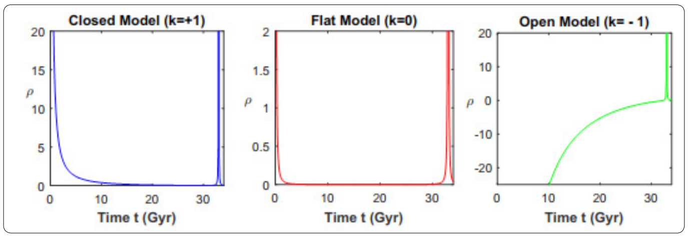

Figure 4. Plot of energy density ρ versus cosmic time t for k=0.097, m=1.6, λ=10 and a_0=1.

Case-2 (ω=1)

In this case the pressure and density are given by

$$\rho = p = \frac{12(kt-m-4)}{2\lambda t^2(kt-2m)^2} - \frac{9k}{2\lambda a_0^2} exp\left(\frac{-4}{m} arctanh\left(\frac{kt}{m}-1\right)\right) \quad (19)$$

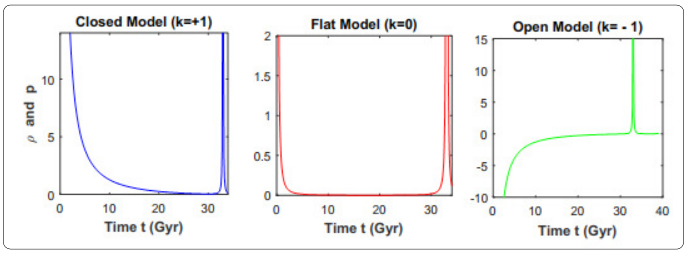

Figure 5. Plot of energy density ρ versus cosmic time t for k=0.097, m=1.6, λ=-10 and a_0=1.

Case-3 ($\omega = \frac{1}{4}$)

In this particular case we have the pressure and density as

$$(12\pi+5\lambda)\rho = (48\pi+20\lambda)p = \frac{24(4-kt+m)}{t^2(kt-2m)^2} + \frac{18k}{a_0^2} exp\left(\frac{-4}{m} arctanh\left(\frac{kt}{m}-1\right)\right) \quad (20)$$

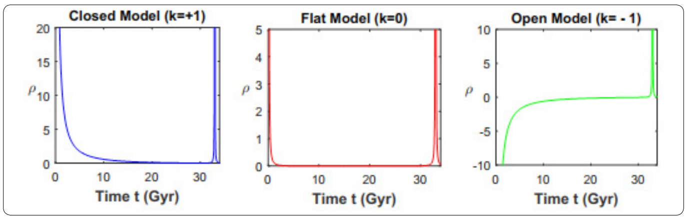

Figure 6. Plot of energy density ρ versus cosmic time t for k=0.097, m=1.6, λ=10 and a_0=1.

From Figure 1 it may be observed that all the models evolve from zero volume, increase with cosmic time and ends at t_{end}≈33 *Gyr*. From observational data we know that the model attains big rip singularity at t_{rip}≈35 *Gyr* (Caldwell [42]). It can be seen that our model is close to the given lifetime for the universe. From Figure 2, it is clear that the Hubble parameter H diverges at the beginning and at the end (i.e., at t_{end}=3 Gyr) of the universe. Figure 3 gives the time varying behavior of the deceleration parameter which exhibits transition of the universe from decelerated phase to the accelerated phase. In fact this happens at t≈6.2 Gyr and the present (i.e. at t=13.7 Gyr) value of the deceleration parameter is q=-0.72 which confirms the observational results of modern cosmology (Cunha [43]). Figures 4-6 show that the energy density of the fluid, in all the above cases, diverge at the beginning and at the end of the universe. Also, we can observe that our models end with a big rip singularity. We notice from these figures that the open models (k = -1) are not possible because of the fact that the positivity condition of the energy density of the fluid is violated in all the cases.

Conclusions

This paper is devoted to the discussion of non-Ricci, non compact FRW type Kaluza-Klien cosmological models in the presence of perfect fluid source in *f (R, T)* gravity. The field equations of the theory are solved using a linearly varying deceleration parameter which includes Berman's law [36] that yields constant deceleration parameter models. We have presented FRW five dimensional cosmological models in *f (R, T)* gravity. Cosmological parameters corresponding to dust, stiff fluid and radiating models are determined and their physical behavior is studied. It is observed that the spatial volume of the universe starts with a big bang at t=0 and ends with a big rip singularity. The energy densities of all the models diverge both at the beginning and at the end of the universe. The models exhibit a smooth transition from early deceleration to present accelerated phase of the universe. It is observed that the models and their physical behavior confirm the recent cosmological data (Caldwell [42]; Cunha [43]; Akarsu and Dereli [38]).

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